Article

Making Sense with Manipulatives: Developing Mathematical Experiences for Early Childhood Teachers

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Abstract

This paper is premised on the fact that math can be an important tool in helping people make sense of the world.[1] Math offers a unique and particular lens, helping people to focus on a range of characteristics from shape and amount to the relationship between the general and the particular. To promote math as a tool for making sense, early childhood math instruction ought to teach it in a manner that helps children make sense of mathematical concepts.[2]

Specifically, I argue here that manipulatives are often brought into the early childhood classroom to promote "hands-on" learning without facilitating making sense. Taking a mixed-methods approach, I move between philosophical analysis to qualitative research to illustrate specific criteria promoting making sense in math education. Building primarily on the philosophy of Maurice Merleau-Ponty, I first define what I mean by "making sense." John Dewey's writing about math education and experience provides a framework for making sense with manipulatives. I then focus on how preservice teachers can teach math to young children in a manner that makes sense. I also share how I changed my instruction using criteria established by early childhood math educators Angela Giglio Andrews and Paul R. Trafton.[3] I conclude by arguing that a math education that makes sense is both a democratic right and necessity.

Living without Math

Longtime teacher and teacher educator Patricia Carini argues that numbers provide one of the many crucial tools that humans use to make sense of their world.[4] In making this claim, Carini retells an extended "Number Story" from Alfred North Whitehead in which a squirrel moves her three children "one by one" to a new location.[5] As Whitehead recounts:

> when the mother had placed them on a rock outside, the family group looked to her very different from its grouping within the nest. She was

> vaguely disturbed, and ran backwards and forwards two or three times to make quite sure no young squirrel had been left behind. She was unable to count. . . . All she knew was that the vague multitude on the rock seemed very unlike the vague multitude in the nest. Her family experience lacked the perception of the exact limitation imposed by number. As a result she was mildly and vaguely disturbed.[6]

I am haunted by the poignancy of the squirrel lacking the tools that would allow her to verify that she had all her children. I also, frankly and somewhat embarrassingly, see a resemblance between my younger self and the squirrel.

As I often disclose to my undergraduate students, for most of my life math made no sense to me. I went through the motions to pass math courses with grades ranging from D+ to A-. Because math in school was a source of stressful confusion, I avoided it in daily life. Much like the squirrel, I approximated where number was concerned. For example, knowing that I had five people coming to dinner, I would carry plates to the table without counting them. I would then match the plates to the seats and, if necessary, go back and forth between shelf and table until I had the corresponding amount. Along similar lines, getting on the subway once, a friend explained how she always entered the fifth car so that she could get off at the right spot in her station. I was shocked. I had always treated my location upon arrival as a kind of unpredictable fate. Even aware of her strategy, I still got on random cars but hoped I'd magically land in the right exit spot.

Unfortunately, my avoidance of math is not unusual. Math tends to be taught in a manner that occludes making sense. As I did initially myself, many students come to see math as an exercise absent of meaning.[7] As Jo Boaler writes of her extensive studies of math education in American and British schools, children often perceive math as a senseless classroom activity, "a strange sort of code," that is inapplicable outside the "boundaries" of school.[8]

My relationship to math changed entirely when I began teaching first and second grades. Being an elementary school teacher and responsible for others, I was very concerned about my negative relationship with math. As I learned math so I could teach it, the world opened up with increasing nuance, efficiency, and sense.

Theoretical Framework: To Make Sense

Constructivist math educators have used the phrase "make sense" as a central tenet of math education.[9] Reading this literature as I prepared my class, I was drawn to that phrase. As a philosopher of education, before adapting the term as central to my own instruction, I first investigated whether the words accurately portrayed my intentions.

Based on my investigations, to make sense in math education is contrasted with education in which children are simply going through the motions. In her studies, Boaler finds that students often see math as a series of arbitrary steps to be memorized without comprehension. In contrast, as articulated by Andrews and

Trafton, “When children make sense of mathematics, they develop deep understanding of important ideas. This means making connections with their informal mathematical knowledge and making connections among mathematical ideas.”[10] Making sense, therefore, is a pulling together of both experiences and concepts.

Breaking down the phrase into “making” and “sense” highlights the respective association with creation and the senses. The senses are not simply felt; something is “made” of them. This speaks to the works of foundational progressive educators like Maria Montessori, John Dewey, and Alfred North Whitehead, who place the senses at the center of the learner’s development.[11] Each argues that the child learns as she experiences the world physically. Yet, for each of these thinkers, simply sensing is not enough. One must “make” or construct meaning from the sensations.

To further articulate this key relationship between the senses and comprehension, I turn to Merleau-Ponty. Two elements of his thinking are particularly helpful here. The first is that sensation and more abstract thinking require each other to create meaning. For Merleau-Ponty, making sense comes from drawing together a host of sensations with prior knowledge. He illustrates this point with the example of a cube. The viewer does not actually “see” the whole cube. The mind, having perceived a cube before, fills in the “hidden” and “distorted” surfaces to create the whole.[12]

The second component I want to emphasize from Merleau-Ponty is that making sense requires an integration of sensations. He argues that “perception of the whole is more natural and more primary than the perception of isolated elements.”[13] Put otherwise, he argues that the perceiver applies an “outline” to sensations that allows for comprehension.[14] The mind fills in the outline of the cube even if it only perceives a small section of it. In a helpful illustration, Merleau-Ponty calls on the folktale *The Blind Men and the Elephant*. In this tale, each man touches a different section of the elephant. Without the full picture, each assumes that the section he touches is the whole and, in doing so, incorrectly applies prior knowledge. For example, touching the tale, one man perceives a string. When the men pool together each of their sensations, they figure out that the object is an elephant.[15]

Additionally, perceptions do not come separately as “a sum of visual, tactile, and audible givens,” but, instead, “I grasp a unique structure of the thing, a unique way of being, which speaks to all my senses at once.”[16] Whereas isolating particular elements is disorientating, perceiving the whole demands a range of sensations. Using film as another example, Merleau-Ponty argues that meaning is constructed through sounds, images, words, and pacing among other elements.

The Process of Making Sense in Early Childhood Math Education

In *The Psychology of Number and Its Application to Methods of Teaching Arithmetic*, James McLellan and John Dewey make a similar claim to Merleau-Ponty, arguing that mathematical understanding demands a link between sensations and generalizations.[17] Just as the blind men initially groped without comprehension, math education tends to offer students a series of disconnected sensations that do not come together to reveal the complete elephant.[18] McClellan and Dewey argue that a common mistake in teaching is to assume that just because the child can perceive "certain physical wholes present to his senses," this translates into "*mental* wholes."[19] Without prior knowledge about elephants, the blind men could not have figured out what they were touching. Similarly, students do not automatically leap between the sensations and mathematical concepts. A child can thoroughly examine a cube without knowing the mathematical properties that make it a cube.

To move from the senses to mental wholes, McLellan and Dewey argue that math understanding requires an integration of "symbols" and "things." When symbols dominate instruction, "mathematics primarily involves mastery of the mathematical rules that govern the manipulation of symbols."[20] When this approach is overemphasized, the child learns to "manipulate" numbers without considering what they represent.[21] Things are "concrete objects" used to represent mathematical concepts.[22] A focus on things has the child manipulating objects without making connections to mathematical rules.[23] The child may use the senses to perceive the object without making any mathematical sense of it. McLellan and Dewey argue that both symbols and things are crucial. Math only makes sense of the world when the two come together.

The dichotomy between symbols and things is currently reflected in what is often referred to as the "math wars." On one side are instructional practices that emphasize the ability to manipulate numerical symbols. Examples of this would be memorizing addition facts through flash cards or learning to add two-digit numbers "by carrying the one." On the other side, the importance of contextual application is emphasized; this is often referred to as "hands-on learning." This can take the form of things often called manipulatives, realistic word problems, and enticing and authentic challenges, such as determining how many cubes it takes to measure a room.[24]

In *Experience and Education*, Dewey firmly rejects binary approaches to education that pit what he refers to as "traditional" modes, which focus on skills, against "progressive" methods, which center on experiential learning.[25] Building on Dewey, I replace the term progressive with the more specific "constructivist" to refer to the educational approach in which the student builds meaning from the opportunity to experiment with materials, ideas, and concepts. Rejecting binaries, I argue with Dewey that math instruction ought to help the child move between sensing the thing and mastering symbols in order to achieve mathematical understanding.[26]

Much criticism among constructivists has been directed at a symbols-based approach taught through worksheets.[27] I fully agree with this criticism. I am also concerned that many so-called constructivist classrooms and curriculums lack attention to skills and content.[28] With a focus on older students, prior constructivist math research tends to focus on finding ways to frame math problems in ways that are more authentic, grounded in examples from life, and therefore compelling.[29] Differing in focus but not in values, as an early childhood educator, I concentrate on manipulatives to showcase how they can be used to support making sense.

Manipulatives

In this paper, a "manipulative" refers to any tangible object used for instruction. Manipulatives can be made for specific purposes, like teaching a particular concept, or they can be part of a larger context, such as cooking or using natural materials. Some educational approaches, such as Montessori, revolve around the use of didactic manipulatives that guide the child toward specific understandings. Other manipulatives, such as unit blocks, are more open-ended in their use and instruction. Because most people learn by moving from the concrete to the more abstract,[30] manipulatives are particularly popular in early childhood work; however, they ought to be a mainstay when people are grappling with something new.

Work with manipulatives should support the acquisition of concepts, but often these connections are not made.[31] Teacher researchers Ruth Shagoury and Brenda Miller Power share the story of teachers who had made cooking central to their first grade math curriculum with the best of intentions. However, after conducting surveys of their students, "A few of the children said that math is 'when you eat food in school.' We hadn't realized how many math activities in kindergarten and first grade involved cooking and eating. One child even said, 'You know you're finished with math when you're full!'"[32] Faced with student confusion, teachers and school systems may abandon the use of manipulatives and return to worksheets, where the connections to math seem more apparent. Thus, the math wars continue as people ricochet between ineffective approaches.

Experiences with Manipulatives

Simply exposing children to manipulatives does not ensure that learning will happen. As noted above, making sense requires drawing together sensations with prior information. Dewey's philosophy of learning from firsthand experience is helpful in accounting for the mental act that takes place as one draws together sensations to learn.[33] According to Dewey, an experience consists of two key criteria: trying or undergoing and reflecting upon both the actions and the results.[34] In this paper, the broadly applied term reflection[35] is used narrowly to refer to the act of reviewing doing and undergoing with an eye toward expanding understanding.

When doing or undergoing, one's senses are bathed in an activity. The necessity of reflection as part of experience is often overlooked in discussions of Dewey in early childhood scholarship.[36] In many classrooms, "mere activity[37]" with manipulatives occurs because the environment does not support reflection.[38] In an apt criticism of simply exposing children to manipulatives, Dewey writes, "There are hundreds of leaves in which the bird builds its nest, but it does not follow that the bird can count."[39] As the teachers discovered with their young cooks, simply doing or undergoing does not ensure that one comes to a deeper understanding.

When a teacher creates an environment in which the student engages in doing, undergoing, and reflecting, the experience guides the student toward making sense.[40] For example, if a child is doing a shape puzzle and does not make the connection between the shape of a piece and the shape of the hole, every time she does the puzzle, she may keep pushing pieces at the holes until they slide into the right one. The child has not learned something new about shapes if she continues to approach the puzzle in the same haphazard way. Another child may try to put the piece in, independently reflect on what isn't working, and then make a more informed attempt. In other words, what a child needs to make sense differs among them. In the following section, I will demonstrate criteria that help teachers create an environment rich enough that a variety of children can make sense of math.

Using Manipulatives to Make Sense of Math with Preservice Teachers

The Psychology of Number and Its Application to Methods of Teaching Arithmetic attempts a marriage of philosophy and practice. To this end, each chapter includes "educational applications," a section that delves extensively into concrete teaching suggestions.[41] Noting that the book has been largely ignored, Kurt Stemhagen brings our attention to the text primarily for its philosophical implications. He writes, "Dewey, did, in fact, forward a clear, distinct and fundamentally original philosophy of mathematics education."[42] While asserting that McClellan and Dewey contribute "interesting . . . methods of mathematics teaching,"[43] Stemhagen leaves analysis of these methods to others.

Educators frequently struggle to connect philosophy with methods. Likewise, philosophers struggle to translate their thinking into viable practice. Further, many administrators and teacher educators are unclear about how to improve teachers' practices.[44] To this end, I turn now from analysis of the philosophical to methods. As a teacher educator and former elementary school teacher, I found McLellan and Dewey's suggestions for practice sound but not particularly accessible. To address what I see as an unfortunate disconnect between their philosophy and compelling methods, instead of deconstructing what I see as lacking, the remainder of this article provides methods that I hope complement their philosophical conclusions.

Self-Study

Math educator and teacher researcher Magdalene Lampert writes:

> The single teacher I study here is myself. Like all teachers, I take a particular approach to teaching, and this book is also a study of that approach

> . . . . I do not attempt to prove that teaching with problems "works." I explain what kind of work is involved in doing it in an ordinary classroom in relations with students and subject matter.[45]

As with Lampert, I illustrate how I revised instruction to help my students make sense of and therefore prepare to teach mathematical concepts with manipulatives.

For this purpose, I draw on five iterations of a required math methods course I taught for undergraduate early childhood majors. My data consists of daily plans, detailed notes taken during class, annotated syllabi, teaching journals, student work, and the books, articles, and manipulatives I used in class. My course was structured around several key concepts: number sense, geometry, algebraic thinking, and measurement. I typically spent two weeks on geometry and measurement, three on number sense, and three on algebraic thinking. While I divided the course into separate concepts, I emphasized that there was significant overlap between them. Of note, number, algebraic thinking, and geometry arguably represent different ways of knowing within math. As McLellan and Dewey point out, measurement provides the purpose for number, and the two cannot be disconnected.[46]

The time spent on a topic was influenced by the complexity of the content, my experience with prior classes, and the needs of a particular class. For example, in a semester where I had only a few students planning to teach in an elementary school setting, we spent less time on methods to support double-digit addition and subtraction. We spent more time on helping infants and toddlers have opportunities to experience change and cause and effect.

Four iterations of the course were designed for preservice undergraduates. One iteration included both in-service and preservice students who had returned to college after earning an associate's degree. The courses for both populations shared many of the same activities and readings, but they were slightly modified for the students with the associate's degrees who had more teaching experience. Students were almost exclusively female and from lower income backgrounds. Most grew up in the rural state where the college is situated. Many of their parents worked in trades like carpentry, plumbing, car repair, fishing, and childcare. Some of the students intended to teach in elementary schools, whereas others were preparing to work in preschools or daycares. Students typically took the math methods course in their junior or senior year, having previously taken at least two math content classes.

Though an experienced teacher, I was new to teaching undergraduates and working with a rural population, and I had no formal education in teaching math methods.[47] Like Lampert, Donald Schön argues that we can learn about a practice by hearing how another practitioner makes decisions. As we become more experienced, Schön found that subtle decisions can seem automatic. In this way, a practitioner's reasoning can become obscured as she becomes more fluent at a task.[48] By exploring my instruction at this stage, I balance experience in teaching with the awareness of minute decisions that comes from being relatively new to the particular work.

Finally, it is important to note that I do not seek to prove that my approach to teaching with manipulatives "works." In fact, in focusing on the newest area of my teaching, I intentionally position myself as a nonexpert. Because seeing a practitioner practice and then hearing her explain her thinking can provide powerful pedagogical insights,[49] I hope that in reading about my teaching readers will take this article as a conversation starter between colleagues about how I connected philosophy and practice when teaching with manipulatives.

Students' Background

Primarily the recipients of a symbols-focused pedagogy, students critiqued previous math experiences in which they completed worksheets and were taught through lecture. Many noted that they had worked with manipulatives as very young children. By their accounts, this work was fun but was rarely connected to math concepts. Because the work with manipulatives wasn't grounded conceptually, students struggled considerably as soon as manipulatives were replaced with formulas. Most students entered the course with a bias toward teaching through things, describing themselves as "hands-on" learners and complaining, at least initially, about readings and lectures. Reflecting the disconnect in their education between symbols and things, many of my students did not readily make connections between manipulatives and math concepts.

The majority of my students portrayed their learning in school as a series of disconnected facts and activities. For the first assignment, students wrote a biographical poem about prior math experiences. Typically, no more than three out of twenty students identified as liking math. A higher percentage recounted positive early childhood experiences that turned sour in later elementary school and in middle school. Each semester, a few students described positive math learning that occurred outside of school. For example, almost every semester, a student wrote warmly about learning math when assisting a family member in a building project.

Longtime English teacher Pat Schmidt writes, "Lionel Trilling made a crucial statement about what may influence teaching when he said, 'The experience of the teacher proposes the possible experience of the student' (Trilling, 1970)."[50] Learning through experience, the teacher is better prepared to help students learn through experience as well. Not only was I concerned about my students' disconnect from math, but I also worried that, lacking this connection, they would pass this alienation on to the next generation. To avoid this, my students needed their own meaningful mathematical experiences with manipulatives if they were to support children's development.[51]

First Iteration of the Course: Disconnect between Symbols and Things

My first semester, I introduced different manipulatives in every class. I emphasized that manipulatives supported deeper understanding of mathematical concepts.

That manipulatives tended to be more fun to work with was an added bonus, not our primary purpose. My focus instead was on promoting manipulatives as a key tool in early math instruction. Students were easily convinced that manipulatives should be used with young children. In fact, they seemed to bring this value to the course. The challenge, as I subsequently discovered, was helping them deploy manipulatives to make sense of math.

In a final take-home exam, students were asked to describe activities they thought would be beneficial and to explain the concepts each supported. Students were assessed on their ability to choose activities, explain the implementation, and articulate the math that could be learned through the activity. In their exams, students embraced using manipulatives, but many remained unclear on the mathematical learning that a given manipulative supported. In explaining their choices, many emphasized the importance of activities being fun and hands-on without making connections to the more abstract math concepts. Exams showed confusion over the mathematical purpose behind a particular activity. Though directed to be specific about the math learning, many were vague. For example, an activity might be classified as teaching "number sense" without indicating what specific elements of number sense would be the focus. Activities were not always suited to the developmental stage for which they were intended. For example, problems that required counting large amounts, thereby working on grouping, were offered as introductory number activities for children who had not yet mastered one-to-one correspondence.

As I looked back on the semester, I felt we had been constantly rushing. In each class students tried many activities. The primary texts were also full of additional activities.[52] My goal was to share a plethora of perspectives and materials so students could find what resonated with their own teaching styles. Students seemed to interpret the inundation as my valuing abundance. This contributed to some students' demoralizing belief that math instruction required far more activities than they could hope to pick up before entering the classroom. Though encouraged to use classwork and readings, many pulled elaborate and confusing activities from websites. Instead of focusing on what made activities successful, they amassed as many activities as possible.

Changing the Curriculum to Help Students Make Sense

Rushing from activity to activity, students spent the semester doing and undergoing without enough reflection to draw out the concepts. To help my students engage with math as a means of making sense, I shifted what we were doing in class. I believed that if, through working with manipulatives and reflecting, students experienced math as making sense, this would be a far more effective lesson than telling them that it was important, as I had done the first semester.

Simply engaging in activities and reflecting is easier said than done. Andrews and Trafton describe the learning environment in Andrews's kindergarten as one in which the children "come to expect math learning to be a sense making experience and therefore they are willing to spend a great amount of time on challenging problems and tasks."[53] Helpfully, Andrews and Trafton provide five criteria of an environment that makes sense. By their estimation, children should have:

1. Opportunities to take ownership of a task
2. Sufficient time to work on tasks
3. Many opportunities to reflect and communicate
4. A rich variety of tools
5. A teacher with a flexible notion of her role[54]

As I will explain below, these criteria help to ensure that the doing, undergoing, and reflecting involved engage the senses and incorporate mathematical concepts. For the remainder of this article, I will provide examples of how I met Andrews and Trafton's criteria as I guided college students' work with manipulatives.

1. Taking Ownership of a Task

In each chapter, Andrews and Trafton describe a different problem that Andrews's kindergarten students take it upon themselves to solve.[55] In one chapter, the children determine the number of bus seats needed for a field trip.[56] In another they look for patterns in the 100s chart.[57] For these children math is not an exercise but rather a tool used to solve problems of interest. They use math to make sense of a situation and, in doing so, the math itself makes sense to them. In taking ownership of the task, the children also take ownership of learning the math involved in the task.

A key to letting students take ownership of the task is providing authentic opportunities for problem-solving.[58] The term "authentic" is often assigned to utilitarian problems that can relate directly to situations students face outside of the classroom.[59] Figuring out how many bus seats were needed for a field trip is an example of such a problem. These problems have considerable merit, but I find direct applicability too limiting. Instead, I measure a problem's authenticity not by its real-world applicability, but by whether it raises interesting questions for the problem-solver. Finding patterns in the 100s chart lacked an immediate tangible use, but it inspired the children.

Ownership helps students tailor learning to their particular needs and interests.[60] To accommodate student interest, I brought to class a few manipulatives that supported the concept we were working on that week. I'd set a length of time (as discussed in the next section) and have students choose what they wanted to work with, with whom, and for how long. Sometimes students were required to try each material. Other times, they had the option of working with as many materials as they liked. For example, when studying sorting, I supplied a range of manipulatives, including buttons, variously colored plastic animals, and pictures of ships.

The instructions were to sort from at least two sets of materials. Students chose the categories to sort by and then determined how to organize and share results. Challenges emerged that forced students to engage more deeply with math. For example, when sorting buttons, some did not fit into obvious categories. Students had to think carefully about how they would graph these results.

2. Sufficient Time

To make the mathematical "abstraction and generalization" that McLellan and Dewey argue are crucial to mathematical development,[61] students need extensive time to explore manipulatives.[62] A cursory look affords only a limited view. In contrast, more time provided results in more potential for deeper connections. The textbooks tended to introduce activities and concepts at a rapid pace.[63] Contemplating including less, I was intimidated about breaking from the style of those highly regarded in the field. New to college teaching and math education, I worried that by engaging in fewer activities we would miss something essential. On the other hand, when I taught elementary school, I did much more when I covered less. I resolved to slow down so that we could carefully process what we were doing.

Initially, I ended a work period when most students slipped into conversations unrelated to math. I found however that some students would frequently lose interest early, only engaging superficially with materials. Consequently, I frequently set a minimum amount of time. Instead of redirecting side conversations, I told students it was their responsibility to stay engaged. We also brainstormed how one could reengage once the initial interest was lost. As students took more responsibility, the lengthy work periods led to deeper work. At the end of the semester, one student reported that she found keeping herself engaged a challenging but particularly valuable part of the course.

3. Many Opportunities for Students to Reflect and Communicate

As noted previously, a key component of experience for Dewey is reflection. Following in this tradition, math educators have argued that reflection and attending to the reflection of others support mathematical understandings.[64] My first semester, prompts tended to be open ended. I was imprecise about the kinds of connections I hoped students would make. When asked to reflect, many students shared that they enjoyed or disliked activities without making connections with the math concepts.

I assumed that my students would see the math simply by being exposed to the materials. Yet, as Dewey points out, the presence of the leaves does not ensure that the bird can count.[65] Subsequently, I gave students prompts that guided connections between activities and specific concepts. Students articulated why they chose to use particular manipulatives for every lesson plan they developed. They also had to identify the mathematical concepts the activity was intended to support. Students were required to be very specific. For example, because the ability to count requires a variety of subskills (e.g., rote counting, one-to-one correspondence, identifying numeric symbols, subitizing), students had to specify what elements of counting an activity addressed. Some reflection happened informally during class. Students were also given weekly assignments involving math-related activities with follow-up prompts.

According to Merleau-Ponty, in making sense we draw on both our personal and cultural backgrounds.[66] To tap into a wider range of experiences, I asked students to explain their process for problem solving. Students could not share the answer until we had multiple strategies on the board. They were encouraged to work together. I also often had students interrupt their work and walk around the room to see what others were doing.

4. A Rich Variety of Tools

Merleau-Ponty emphasizes that we make sense by drawing from multiple senses.[67] Similarly, in math, students make sense of a concept by approaching it through a variety of manipulatives. Given that students learn differently and have different interests, a range of manipulatives that address the same concept is necessary.[68] When sharing responses to a manipulative, some of my students would say they found it very helpful and others expressed resistance. For example, many students had warm memories of using wooden-pattern blocks. They reported enjoying the feel and the ways the blocks fit together. Some students complained about plastic-pattern blocks, saying that they didn't feel as good in the hand and were harder to grasp and manipulate. One semester, a student suggested that the clicking noise the blocks made when placed on the table would distract some of the children she currently worked with. Students also articulated the importance of using manipulatives that a learner finds appealing.

After students had explored a manipulative, they would brainstorm activities using it and the concepts it would support. A tool like unifix cubes, for example, could be used for counting, grouping, measuring length and volume, building three-dimensional towers, solving algebraic equations, and many other activities. Students found that some manipulatives, such as shape sorters, most board puzzles, and many of the number games on the market, are limited to a particular task. These manipulatives can thus be very helpful for teaching a specific skill to a particular child. The disadvantage is that when a whole class uses the same limited-purpose manipulative, the needs of only a few children are met. Such manipulatives also lose their value once a child has outgrown the task.

Other times, we started with the concept and students brainstormed the manipulatives and corresponding activities to teach it. For example, when we studied counting, students brought to class a game and explained exactly what elements of counting it supported. In one class, students could choose any material found in the classroom (with an attached math materials closet there was an abundance to choose from) to measure the length of the floor. They tried unifix cubes, hands, textbooks, and much more. As a class, we then discussed how each manipulative worked. Students noted that measuring the room in bodies would help young children get a general sense of measurement, but it wasn't very accurate. They also noted that small objects like unfix cubes would be hard for beginning counters because so many were required.

5. A Teacher with a Flexible Notion of Her Role

As teachers, we cannot know ahead of time what mathematical connections a child has already made. We also cannot be sure what materials and activities will be beneficial. With this in mind, my first role was as an observer, taking notes as students spoke and worked and collecting student work and reflections. As I explained, I used this data to inform my instruction. This meant that, each semester, pace and activities changed. Sometimes I changed a lesson midsession based on my observations or student feedback. When I made these changes, I explained my reasoning.

Students shared the role of teacher by observing each other and providing feedback. They also practiced being observant teachers by studying children's work, watching videos, and observing children in person. One semester, students did a math lesson at a local school. Another semester, students gave a demonstration lesson to our class.

Helping students make sense demanded that I employ a range of pedagogical approaches. While this article primarily addresses our work with manipulatives, among many activities, students also solved problems on the board, occasionally filled out worksheets, went on scavenger hunts for particular math-related items, watched videos, heard short lectures from me and guests, and listened to children's books being read. We discussed the different roles that I played in the classroom and the effectiveness of different approaches.

An Education That Makes Sense

Having now taught this course four times drawing on Andrews and Trafton's criteria,[69] I have found that their categories helped me to promote meaningful connections between manipulatives and concepts. I measured success by students' improvement in a variety of informal and formal assessments. As an example of an informal assessment, students began to question whether lessons they saw in classrooms and found online made sense. For example, after using geoboards[70] in class to make a variety of shapes, many students commented that they had never seen the purpose of this tool before. They expressed concern for how it was being used in the classrooms in which they had been. With the focus on making sense, students' final products were far superior than they were at the end of the first semester. In contrast to the trepidation expressed at the start of the semester, in final reflections the majority of students articulated a new confidence in developing and implementing math instruction for young children. Most also expressed a new confidence in math.

Andrews and Trafton critique making sense as largely treated as an end in itself.[71] Building on their work, I instead argue that making sense of math is an important means to a larger end. In his seminal work, *Democracy and Education*, Dewey argues that schooling is necessary because the modern world is too complex for children to simply learn through apprenticeship.[72] The child joins into a culture that demands literacy in the traditional academic disciplines (reading, writing, history, math, etc.). Participating in modern society requires that the learner have basic math skills at hand.[73] Without number, the squirrel could not guarantee that her family was protected. Such confusion is all the more poignant on the human scale. Further, math is not only a series of useful skills; it also provides a way of making sense of the world.[74] For example, math work can help students develop problem-solving strategies, engage in part-to-whole thinking, work from the particular to the general, and spatially and numerically orient themselves.

Dewey laments that many educational experiences “limit” students’ “power of judgment and capacity to act intelligently in new situations.”[75] In reading Dewey, Stemhagen writes that he “was concerned that an emphasis on efficiency and the prescriptive, mechanical treatment of education would lead to Platonic class divisions and enslave individuals who could not understand or control the aims of their learning and work.”[76] Without comprehension, students are beholden to others’ decisions.[77] To act effectively and ethically requires that we understand our environment. [78] Put otherwise, the experience of the world as nonsensical is disempowering. In exposing children to math (and other disciplines) in a manner that makes no sense, we alienate them from the culture in which they must operate. Therefore, making sense of math, as well as other academic disciplines, is crucial to being able to participate in the kind of decision-making a democratic society requires.[79]

Finally, though nonsensical schooling is a challenge for all communities, it is a particular injustice to students from underprivileged backgrounds. Despite Dewey’s call for school as the place for academic learning, many children acquire much of their academic competency outside of school.[80] Schools that house children with greater academic, physical, emotional, and economic needs tend to offer more rote learning and fewer opportunities for working with content in a manner that promotes sense.[81] In this way, students who come from families that are already disempowered are more likely to leave school without assimilating the academic epistemologies of the dominant culture. Though others have connected math education to promoting democratic citizenship, the emphasis tends to be on ensuring that future math educators hail from a diverse population, that a robust math education is available to all students,[82] and that students can learn math concepts through exploration of real-world problems.[83] While worthy endeavors, the way math instruction is taught strongly influences its potential to support democratic thinking. Because those with power in our society tend to draw on academic subjects with fluency, students without fluency are estranged from power.

To conclude, I reiterate that my purpose is not to advocate for the value of math as opposed to other subjects. Nor do I seek to elevate academic learning over other ways that people make sense of the world. Instead, academic learning, with math education as my focus, is one of the many relevant ways in which people make sense. Therefore, it must be taught in a manner that makes sense itself. Integrating Andrews and Trafton's criteria with a philosophical understanding of making sense helped me to teach about manipulatives in a manner that made more sense to my students. As an early childhood instructor in math education, I hope that my students will leave class passionate and knowledgeable about math content. Even more pressing, I want them to leave my class with the faith that math can and ought to make sense.

Notes

1. Bülent Atalay, *Math and the Mona Lisa: The Art and Science of Leonardo Da Vinci* (New York: Smithsonian Books, 2006); Jo Boaler, *What's Math Got to Do with It? How Parents and Teachers Can Help Children Learn to Love Their Least Favorite Subject* (New York: Penguin Books, 2009); Sally Moomaw, *Teaching Mathematics in Early Childhood* (Baltimore: Brookes Publishing Company, 2011), 1; Frances Stern, *Adding Math, Subtracting Tension: A Guide to Raising Children Who Can Do Math* (Reston, VA: National Council of Teachers of Mathematics, 2011), vii; Alfred North Whitehead, *The Aims of Education and Other Essays* (New York: Free Press, 1967).

2 Angela Andrews and Paul R. Trafton, *Little Kids—Powerful Problem Solvers: Math Stories from a Kindergarten Classroom* (Portsmouth, NH: Heinemann, 2002); Eugene Geist, *Children Are Born Mathematicians: Supporting Mathematical Development, Birth to Age 8* (Upper Saddle River, NJ: Merrill/Pearson, 2009); Stern, *Adding Math, Subtracting Tension*.

3. Andrews and Trafton, *Little Kids*.

4. Patricia F. Carini, *Starting Strong: A Different Look at Children, Schools, and Standards*, The Practitioner Inquiry Series (New York: Teachers College Press, 2001).

5. Ibid., 190.

6. Whitehead, *Modes of Thought*, 106–07, quoted in Carini, *Starting Strong*, 190.

7. Jo Boaler, *What's Math Got to Do with It*?

8. Ibid., 76.

9. Boaler, *What's Math Got to Do with It?*; Andrews and Trafton, *Little Kids.*

10. Andrews and Trafton, *Little Kids*, 4.

11. John Dewey, *Democracy and Education: An Introduction to the Philosophy of Education* (New York: Free Press, 1997); John Dewey, *Experience and Education*, The Kappa Delta Pi Lecture Series (New York: Simon & Schuster, 1997); Maria Montessori, *The Montessori Method* (Mineola, NY: Dover Publications, 2002); Whitehead, *The Aims of Education*.

12. Maurice Merleau-Ponty, Hubert L Dreyfus, and Patricia Allen Dreyfus, *Sense and Non-Sense* (Evanston, IL: Northwestern University Press, 1964), 50.

13. Ibid., 49.

14. Ibid., 13.

15. Maurice Merleau-Ponty, *The World of Perception*, Routledge Classics (London: Routledge, 2009), 59–60.

16. Merleau-Ponty, Dreyfus, and Dreyfus, *Sense and Non-Sense*, 50.

17. James Alexander McLellan and John Dewey, *The Psychology of Number and Its Applications to Methods of Teaching Arithmetic* (Charleston: BiblioBazaar, 2009).

18. Boaler, *What's Math Got to Do With It?*

19. McLellan and Dewey, *The Psychology of Number*, 29.

20. Kurt Stemhagen, "Toward a Pragmatic/Contextual Philosophy of Mathematics: Recovering Dewey's Psychology of Number," *Philosophy of Education 2003*, 440.

21. Ibid.

22. Ibid.

23. McLellan and Dewey, *The Psychology of Number*.

24. Boaler, *What's Math Got to Do With It?*

25. Dewey, *Experience and Education*.

26. McLellan and Dewey, *The Psychology of Number*.

27. Boaler, *What's Math Got to Do with It?*

28. Ibid.; David K. Cohen, "A Revolution in One Classroom," *Educational Evaluation and Policy Analysis* 12, no. 4 (1990): 311–29; Kurt Stemhagen, "Democracy and School Math: Teacher Belief-Practice Tensions and the Problems of Empirical Research in Educational Aims," *Democracy and Education* 19, no. 2 (2011): 1–11; Kurt Stemhagen and Jason W. Smith, "Dewey, Democracy, and Mathematics Education: Reconceptualizing the Last Bastion of Curricular Certainty," *Education and Culture* 24, no. 2 (2008): 25-40.

Ana; Kurt Stemhagen, "Deweyan Democratic Agency and School Math: Beyond Constructivism and Critique," *Educational Theory* 66, no. 1–2 (2016): 95–109.

29. Boaler, *What's Math Got to Do With It?;* Magdalene Lampert, *Teaching Problems and the Problems of Teaching* (New Haven, CT: Yale University Press, 2001). Nadia Stoyanova Kennedy, "Lipman, Dewey, and Philosophical Inquiry in the Mathematics Classroom," *Education and Culture* 28, no. 2 (2012): 81–94. Dov Zazkis and Rina Zazkis, "Wondering About Wonder in Mathematics," in *Wonder-Full Education*, ed. Kieran Egan, Annabella I. Cant, and Gillian Judson (New York: Routledge, 2014).

30. David K. Cohen, "A Revolution in One Classroom"; Geist, *Children are Born Mathematicians*; Janice R. Minetola, Robert G. Ziegenfuss, and J. Kent Chrisman, *Teaching Young Children Mathematics* (New York: Routledge, Taylor & Francis Group, 2014); David A. Sousa, *How the Brain Learns Mathematics* (Thousand Oaks, CA: Corwin Press, 2008).

31. David K. Cohen and Deborah Loewenberg Ball, "Relations between Policy and Practice: A Commentary," *Educational Evaluation and Policy Analysis* 12, no. 3 (1990): 331–38; Cohen, "A Revolution in One Classroom"; Geist, *Children Are Born Mathematicians;* Elizabeth Green, *Building a Better Teacher: How Teaching Works (and How to Teach It to Everyone)* (New York: W.W. Norton & Company, 2015).

32. Ruth Shagoury and Brenda Miller Power, *The Art of Classroom Inquiry: A Handbook for Teacher-Researchers*, rev. ed. (Portsmouth, NH: Heinemann, 2003), 70.

33. Dewey, *Experience and Education*.

34. Dewey, *Democracy and Education*, 139.

35. Carrie Birmingham, "Phronesis: A Model for Pedagogical Reflection," *Journal of Teacher Education* 55, no. 4 (2004): 313–24; Terence H. McLaughlan, "Beyond the Reflective Teacher," *Educational Philosophy and Theory* 31, no. 1 (1999): 9–25; Hugh Munby, "Reflection-In-Action and Reflection-On-Action," *Education & Culture* 9, no. 1 (1989): 31–41.

36. Ysaaca Axelrod, "'We All Is Teachers': Emergent Bilingual Children at the Center of the Curriculum," *Occasional Papers Series*, 35 (2016): 88–97; Lissanna M. Follari, *Foundations and Best Practices in Early Childhood Education: History, Theories, and Approaches to Learning*, 3rd ed. (Hoboken, NJ: Pearson, 2015), 41.

37. Dewey, *Democracy and Education*, 139.

38. Cohen, "A Revolution in One Classroom."
39. Stemhagen, "Democracy, Dewey, and Mathematics Education," 32.
40. Dewey, *Experience and Education*.
41. McLellan and Dewey, *The Psychology of Number*.
42. Stemhagen, "Toward a Pragmatic/Contextual Philosophy of Mathematics," 436.
43. Ibid.
44. Cohen, "A Revolution in One Classroom"; Green, *Building a Better Teacher*.
45. Lampert, *Teaching Problems*, 1.
46. McLellan and Dewey, *The Psychology of Number*, chapter 3.
47. I did have experience as an early child math teacher, had taken math courses during my Masters in education, and had done workshops in math instruction when I taught in the elementary classroom.
48. Donald A. Schön, *The Reflective Practitioner: How Professionals Think in Action* (New York: Basic Books, 1983).
49. Green, *Building a Better Teacher*; Schön, *The Reflective Practitioner*; Donald A. Schön, *Educating the Reflective Practitioner: Toward a New Design for Teaching and Learning in the Professions*, The Jossey-Bass Higher Education Series (San Francisco: Jossey-Bass, 1987).
50. Patricia A. Schmidt, *Beginning in Retrospect: Writing and Reading a Teacher's Life* (New York: Teachers College Press, 1997), 168.
51. Cohen, "A Revolution in One Classroom."
52. Andrews and Trafton, *Little Kids*; Rosalind Charlesworth, *Experiences in Math for Young Children*, 6th ed. (Boston: Wadsworth Cengage Learning, 2012).
53. Andrews and Trafton, *Little Children*, 4.
54. Ibid., 4–5.
55. Andrews and Trafton, *Little Kids*.
56. Ibid., chapter 10.
57. Ibid., chapter 8.
58. Boaler, *What's Math Got to do With It?*; Lampert, *Teaching Problems*; Zazkis and Zazkis, "Wondering About Wonder in Mathematics."
59. Anastasia Brelias, "Mathematics for What? High School Students Reflect on Mathematics as a Tool for Social Inquiry," *Democracy & Education* 23, no. 1 (2015): 1–11.
60. Dewey, *Democracy and Education*, 194.
61. McLellan and Dewey, *The Psychology of Number*, 35.
62. Andrews and Trafton, *Little Kids*; Geist, *Children are Born Mathematicians*.
63. Charlesworth, *Experiences in Math for Young Children*; Moomaw, *Teaching Mathematics*.
64. Andrews and Trafton, *Little Kids*, 6; Moomaw, *Teaching Mathematics in Early Childhood*, 26.
65. Stemhagen, "Democracy, Dewey, and Mathematics Education," 32.
66. Maurice Merleau-Ponty and James M. Edie, *The Primacy of Perception: And Other Essays on Phenomenological Psychology, the Philosophy of Art, History and Politics*, 2nd paperback printing, Northwestern University Studies in Phenomenology & Existential Philosophy (Evanston, IL: Northwestern University Press, 1971).
67. Merleau-Ponty, *Sense and Non-Sense*.
68. Stern, *Adding Math Subtracting Tension*.
69. Andrews and Trafton, *Little Kids*, 4–5.
70. Pegboards that allow you to make 2-D shapes with rubber bands.

71. Andrews and Trafton, *Little Kids*.

72. John Dewey, *Democracy and Education: An Introduction to the Philosophy of Education* (New York: Free Press, 1997).

73. Boaler, *What's Math Got to Do with It?*

74. Carini, *Starting Strong*; Kennedy, "Lipman, Dewey, and Philosophical Inquiry."

75. Dewey, *Experience and* Education, Ibid., 27.

76. Stemhagen and Smith, "Dewey, Democracy, and Mathematics Education," 27.

77. Brelias, "Mathematics for What?"

78. Dewey, *Experience and Education*; Paulo Freire, *Pedagogy of the Oppressed*, 30th anniversary ed. (New York: Continuum, 2000).

79. Stemhagen, "Democracy and School Math."

80. Lisa D. Delpit, *Other People's Children: Cultural Conflict in the Classroom* (New York: New Press, 2006. Distributed by W.W. Norton).

81. Boaler, *What's Math Got to Do with It?*

82. Anita Bright, "Lift Every Voice and Sing," *Democracy & Education* 23, no. 1 (2015): 1–5.

83. Stemhagen and Smith, "Dewey, Democracy, and Mathematics Education"; Brelias, "Mathematics for What."

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